Stochastic Modeling of the Last Mile Problem for Delivery Fleet Planning

by Jay R. Brown and Alfred L. Guiffrida

This paper presents a stochastic representation of the last mile problem that quantifies expected maintenance, regular labor, overtime labor, fuel, and carbon emission costs resulting from different delivery fleet options. The last mile delivery fleet planning model presented herein can be used in a decision framework to evaluate alternative delivery strategies involving fleet size and delivery frequency with information regarding cost, carbon emissions, service levels for available delivery hours, and payload capacity, as well as the transportation capacity needed to meet customer demand and lends itself well to performing what-if analyses.

INTRODUCTION

Environmental concerns are impacting how organizations design, coordinate, and manage their supply chains, and have generated a huge interest in the topic of green supply chain management (Gurtu et al. 2017; Das and Posinasetti 2015; Fahimnia et al. 2015). Srivastava (2007) defines green supply chain management (GSCM) as the integration of environmental thinking into supply chain management. For a gateway into the GSCM literature, the reader is referred to the review articles contributed by Shibin et al. (2016), Wong et al. (2015), Min and Kim (2012), and Sarkis et al. (2011). Managerial interest in GSCM practices is found in a wide cross section of industries. The global scope of this interest is illustrated by recent case study research on GSCM in the construction industry in the United Arab Emirates (Balasubramanian and Sundarakani 2017), meat processing in Italy (Sgarbossa and Russo 2017), mining in Ghana (Kusi-Sarpong et al. 2016), heavy equipment manufacturing in India (Gandhi et al. 2015), rubber processing in Indonesia (Darmawan et al. 2014), and automobile manufacturing in Korea (Lee 2011).

In a climate of enhanced awareness of environmentally sustainable business practices, greater pressure is being placed on organizations to advance sustainable logistics within the operation of their supply chains (Abbasi and Nilsson 2016; Björklund et al. 2016; Golicic et al. 2010). Comprehensive reviews of sustainable logistics have been conducted for various modes of freight transportation, including roadways (Demir et al. 2014), railways (Aditjandra et al. 2016), maritime (Davarzani et al. 2015), air (Teoh and Khoo 2016), and intermodal (Roso 2013). The carbon footprint associated with freight transport across all modes is rapidly becoming a key managerial concern and is especially prevalent in road transportation by transport vehicles in North America and Europe.

The last mile problem (LMP) implies making deliveries from the location (depot/warehouse/transportation hub) where products are maintained in the supply chain to the home address or designated collection point requested by customers. The LMP is a key element of the order fulfillment process within the operation of a supply chain (Bromage 2001; Lee and Whang 2001). While logistics costs vary with population density, product type, package size, and package weight, last mile delivery has shown to incur the highest transportation costs in the supply chain (Chopra 2003). For example, Grackin (2014) reports that the last mile delivery cost for a blouse manufactured in Asia and then delivered and sold to a customer in the United States can range from $4-$11, approximately 28%-78% of the total transportation cost. Because of the increased number and complexity of making last mile deliveries, cost and carbon emissions are becoming critical.
concerns of supply chain managers, and companies have identified that they can save substantial cost, time, and carbon emissions by optimizing last mile delivery.

As e-commerce continues to grow, a greater burden is placed on last mile delivery. Logistics systems, which were traditionally designed to accommodate a single lot delivery of multiple products from one business to another, are now under pressure to deliver high volumes of single parcels from a transportation/warehouse hub to multiple individual customers. With the global B2C (business to consumer) e-commerce market projected to reach $2.3 trillion by 2017, resolving last mile delivery issues will become a major customer service concern for firms (Khanna 2015). Millar (2015) comments that in addition to the cost of transportation, last mile home delivery presents additional managerial challenges as a result of invalid, incorrect, or hard to locate delivery addresses and unaccepted deliveries. Wygonik and Goodchild (2012) show that the routing and scheduling strategy plays a significant role in reducing emissions. The high costs associated with last mile delivery provide an opportunity for companies to achieve substantial cost and carbon emissions reductions through optimal planning and proper execution of a delivery plan.

Determining an optimal delivery fleet for dealing with the LMP, including the number of vehicles, available delivery hours, delivery range, and total freight capacity, is a problem with a solution that varies depending on the customer demand for that delivery cycle. If demand is high, more vehicles, more delivery time, or more freight capacity may be needed. On a day with less demand, the opposite is true. Demand for last mile delivery is typically not constant, and as a result, changes to the fleet operating characteristics may change from week to week or even from day to day. In this paper we present a stochastic model that can be used to aid decision makers in managing the operating characteristics of their vehicle delivery fleet in support of the LMP. The modeling framework is driven by stochastic customer demand and provides managers with metrics on service levels for on-time delivery, vehicle capacity, and driver labor hours. The last mile delivery fleet planning model presented herein can assist decision makers when dealing with uncertainties in delivery fleet planning and aid in finding the optimal delivery fleet strategy based on cost, carbon emissions, payload service level, and on-time delivery service level.

A distinguishing feature of the last mile delivery fleet planning model presented in this paper compared with other fleet models, is the stochastic modeling of travel distances for meeting customer demand in the last mile setting. The stochastic modeling of the LMP for delivery fleet planning allows for: 1) incorporating stochastic demand into the LMP in a varying multiple vehicle delivery environment, 2) incorporating carbon emissions that result from vehicle transportation into the LMP, and 3) the development of a planning model that incorporates uncertainty in terms of expected mileage and labor hours. By modeling LMP travel distances stochastically, the last mile delivery fleet planning model presented herein advances the literature since it improves the quality of information available to plan properly and enhances overall decision making due to the ability to gauge the probabilistic likelihood of the service level and costs associated with solving the LMP.

LITERATURE REVIEW

The development of the stochastic LMP model for delivery fleet planning presented herein draws upon the literature referred to in the operations research literature as “geometric probability” (Larson and Odoni 1981, Ch. 3). These models have been developed to determine the optimal distance traveled from a source (or set of sources) to customers in a geometric shaped demand region. These models can be broken down into two subgroups. Discrete models, which have known customer demand locations, seek to find the shortest total travel distance or most economical way to deliver goods to meet customer demand. With these discrete models, both demand at the known customer locations and travel distances for delivery routes between all locations are deterministic. Continuous models are characterized by a known amount of customer demand within a given demand region; however, the exact locations of the customers within the demand region are not known. Once the customer
demand locations within the demand region are populated, delivery routes with deterministic travel distances are established to service the customers. Customer demand is deterministic and the model seeks to determine an approximate total travel distance as a function of the characteristics of the demand region, such as size, geometric shape of the demand region, and the level of customer demand within the region (Beardwood et al. 1959; Newell 1973; Stein 1978; Daganzo 1984; Vaughan 1984; Jaillet 1988; Stone 1991). Route delivery distances can be measured using the Manhattan ($L_1$) metric, which implies only north, south, east, or west movements are allowed or the Euclidean ($L_2$) metric, which is direct point-to-point regardless of direction. The methodology used in these models has mostly been centered on approximating an average distance to tour all points as opposed to using the distribution of expected distance to capture the true stochastic nature of delivery by transport vehicle in the LMP. The distribution of expected distances is needed to provide management with a service-level-based decision tool for fleet management when meeting customer deliveries.

Langevin et al. (1996) provides a detailed review of the literature on continuous approximation models. These models approximate travel distance based on factors such as region size, shape, and number of deliveries. These models are attractive to supply chain analysts with respect to fleet planning for the LMP since distance is the key driver for evaluating vehicle delivery cost and capacity, determining delivery zones, and estimating the cost of the carbon footprint.

While not addressing the same fleet modeling characteristics and delivery problem found in the LMP, other models in the literature have provided functions for approximating the cost of serving customers under stochastic demand conditions. The research presented herein is not focused on the vehicle routing problem (VRP) since this research does not determine the specific routing of vehicles. The stochastic nature of this research provides an expected travel distance distribution based on the inputs. However, contributions from this area present similarities. The Lei et al. (2012) model can solve a combined vehicle routing and districting problem with stochastic customers. Carlsson et al. (2012) partition a geometric region to assign equitable workloads to vehicles for the stochastic vehicle routing problem.

The problem of optimizing a delivery fleet has been studied from many different perspectives. Jabali et al. (2012), while focused on a deterministic problem, present a continuous time approximation for fleet sizing that they embed in an optimization. Bent and Van Hentenryck (2004) consider the partially dynamic vehicle routing problem with time windows, where some customers are known for planning time while others are dynamic, with the goal of maximizing the number of customers served based on a fixed number of vehicles. Pillac et al. (2013) provide a review of dynamic VRPs that touching on stochastic modeling and vehicle fleet management.

Incorporating carbon emissions into decision-making has been an increasing area of recent interest in both research and practice. Glock and Kim (2015) studied a multiple-supplier-single-buyer supply chain that uses a heterogeneous vehicle fleet for transporting products and found that considering emission cost in the optimization problem may lead to different routing and production policies for the system. In a review of the literature on the LMP, Edwards et al. (2010) directly incorporate carbon emissions ($CO_2$) into the LMP. They introduce carbon footprint analysis to the LMP and compare the level of carbon emissions resulting from online versus conventional shopping for the non-food retail sector. Carbon emission from delivery vehicles was defined by the number of grams of $CO_2$ emitted per kilometer traveled, and the rate of emission was estimated based on secondary technical data sources of vehicle operation. Home deliveries and typical shopping trips were compared based on the aggregate gram weight of $CO_2$ generated during delivery. The findings suggest that home deliveries result in lower carbon emissions.

Zhang et al. (2015) present a low-carbon routing problem that merges the VRP with traditional costs and carbon emissions for a fixed fleet of delivery vehicles. Similar to the last mile delivery fleet planning model presented herein, carbon emission is calculated from travel distance, load, and speed. Soysal et al. (2014) develop a multi-objective linear programming (MOLP) model for a generic beef logistics network problem to minimize total logistics cost and secondarily minimize emissions from
transportation operations. Tang et al. (2015) show that reducing shipping frequency in a periodic inventory review system can reduce carbon emissions with a limited impact on total cost. Govindan et al. (2014) develop a model for simultaneously minimizing logistic costs and carbon emissions in a perishable two-echelon multiple-vehicle location-routing problem. Rigot-Muller et al. (2013) present optimization methods for minimizing total logistics-related carbon emissions for end-to-end supply chain distribution systems using Value Stream Mapping (VSM). These research articles demonstrate that while cost remains a key driver of behavior, reducing carbon emissions can be considered a complementary driver in many cases.

While similarities to certain aspects of this research exist in the literature, none have all the same characteristics. The research herein aims to increase the practicality of delivery fleet planning by creating a decision framework that includes a distribution of expected travel distance, probabilistic service levels, a carbon emissions component, and a multiple vehicle fleet of varying size. Much of the research involving stochastic demand only considers a single delivery vehicle (Ghiani et al. 2012) or rarely a fixed number of vehicles (Bent and Van Hentenryck 2004). Not having the ability to modify the size of the delivery fleet is a serious limitation that the research herein overcomes. Even week to week with changing demand, the ability to adjust the number of delivery vehicles can lower operational costs and improve customer service levels. A modeling environment that accommodates a multiple vehicle fleet of varying size delivering to ever-changing customers provides a greater range of decision options for management in satisfying customer demand.

**STOCHASTIC LAST MILE MODEL DEVELOPMENT**

Brown and Guiffrida (2014) introduced a stochastic last mile framework to model the distribution of expected travel distance based on the number of delivery vehicles, number of customers, and the size of the delivery region. The model assumes a circular demand region with a radius of $R$ surrounding a centrally located depot as the starting point. Demand is considered to be uniformly and randomly distributed, which is supported in a review of continuous approximation models in freight distribution by Langevin et al. (1996). The number of delivery vehicles can vary from one to five vehicles, which results in the demand region being subdivided into $T$ equally sized and shaped sub regions. Each sub region was assigned one vehicle; hence the number of sub regions equates directly with the number of individual delivery tours. Travel distance is measured in miles using the Manhattan ($L_1$) distance metric.

The expected distance traveled per day, $\mu_T$, with standard deviation, $\sigma_T$, is a function of the radius of the demand region, $R$, the number of vehicles, $T$, and the number of nodes (customer delivery points plus the depot), $N$. Table 1 contains the formulas for quantifying the distribution of travel distances employed in this research. Brown and Guiffrida (2014) introduced these formulas in research to calculate the number of customers needed for a last mile delivery service to reach a carbon footprint break-even point with customer pick up. The research herein employs these formulas to capture expected travel distance, but from there a completely new model is introduced to create a decision framework for evaluating alternative delivery strategies involving fleet size and delivery frequency with information regarding cost, carbon emissions, and probabilistic service levels.
Table 1: Statistical Information of Model Equations

<table>
<thead>
<tr>
<th></th>
<th>Mean Equation $\mu_T$</th>
<th>R²</th>
<th>Sig.</th>
<th>Standard Deviation Equation $\sigma_T$</th>
<th>R²</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(0.017 + 1.871\sqrt{N})$</td>
<td>.998</td>
<td>&lt;.0001*</td>
<td>$(0.990 - 0.064 \ln N)$</td>
<td>.528</td>
<td>.0075*</td>
</tr>
<tr>
<td>2</td>
<td>$(1.862 + 1.744\sqrt{N})$</td>
<td>.999</td>
<td>&lt;.0001*</td>
<td>$(1.591 - 0.199 \ln N)$</td>
<td>.832</td>
<td>.0308*</td>
</tr>
<tr>
<td>3</td>
<td>$(2.776 + 1.770\sqrt{N})$</td>
<td>.999</td>
<td>&lt;.0001*</td>
<td>$(1.864 - 0.245 \ln N)$</td>
<td>.871</td>
<td>.0206*</td>
</tr>
<tr>
<td>4</td>
<td>$(3.721 + 1.706\sqrt{N})$</td>
<td>.998</td>
<td>&lt;.0001*</td>
<td>$(2.336 - 0.367 \ln N)$</td>
<td>.861</td>
<td>.0229*</td>
</tr>
<tr>
<td>5</td>
<td>$(4.666 + 1.794\sqrt{N})$</td>
<td>.989</td>
<td>&lt;.0005*</td>
<td>$(2.789 - 0.434 \ln N)$</td>
<td>.940</td>
<td>.0063*</td>
</tr>
</tbody>
</table>

LAST MILE DELIVERY FLEET PLANNING MODEL FORMULATION

Using the stochastic last mile delivery framework, a last mile delivery fleet planning model is formulated to determine the optimal number of vehicles and the optimal number of days to deliver per week subject to the demand distribution, costs associated with the deliveries, the number of hours available per day to make deliveries, radius of the demand region, average vehicle speed in the region, average time spent per delivery stop, and vehicle payload capacity.

The following assumptions are adopted. Distance is measured in miles using the Manhattan ($L_1$) distance metric. A fleet of vehicles is available for use from a central depot. In this example, the vehicles are contracted so the model does not specify the acquisition cost of each vehicle, but this could be explored over a long-term planning horizon if appropriate for the company. The company in this scenario has some flexibility in its weekly delivery plan. For example, a home improvement store could choose to deliver just three days per week with one vehicle if demand was low, or it could be delivering seven days per week with five vehicles if demand was high.

Notation where $T$ and $D$ are the decision variables is as follows.

\begin{align*}
C_E &= \text{CO}_2 \text{ emission cost per gallon of fuel} \\
C_F &= \text{fuel cost ($/gallon of fuel$)} \\
C_M &= \text{maintenance cost ($/mile$)} \\
C_W &= \text{labor wage rate per hour} \\
D &= \text{number of delivery days per week} \\
H &= \text{feasible delivery hours per day} \\
H_W &= \text{regular time labor hours available per week} \\
P &= \text{wage premium for overtime (percent above regular wage rate)} \\
N &= \text{the number of nodes including the depot.} \\
n &= N - 1; \text{the number of customers or delivery points.} \\
R &= \text{the radius (in miles) of the circular demand region} \\
V &= \text{average speed of delivery vehicles in miles per hour} \\
T &= \text{number of delivery vehicles} \\
A &= \text{average time spent at each stop or customer location in hours} \\
TC(T, D) &= \text{total cost as a function of } T \text{ and } D \\
m^*_T &= TV[H_W - DA(N - 1)]; \text{regular time delivery miles available per week} \\
m_D^* &= TV[H - A(N - 1)]; \text{total feasible delivery miles per day} \\
L &= \text{beginning vehicle payload (lbs)} \\
\beta_0 &= \text{miles per gallon (MPG) of an empty vehicle} \\
\beta_i &= \text{loss of MPG for each lb of payload } (\beta_i < 0) \\
\beta_i &= \text{fuel used while idling (gallons per hour)} \\
A_B &= \text{percent of time vehicle is left running at delivery stops } (0 \leq A_B \leq 1) \\
S &= \text{successful delivery rate } (0 \leq S \leq 1) \\
C_T &= \text{traffic congestion factor } (0 \leq C_T \leq 1) \\
E_F &= \text{amount of } \text{CO}_2 \text{ emitted per gallon of fuel}
\end{align*}
Fleet Planning

\[ \mu_{LW} = \text{mean customer order weight in lbs.} \]

\[ \sigma_{LW} = \text{standard deviation of customer order weight in lbs.} \]

\[ \mu_{LV} = \text{mean customer order volume in ft}^3 \]

\[ \sigma_{LV} = \text{standard deviation of customer order volume in ft}^3 \]

\[ L_w^* = \text{weight capacity of a vehicle in lbs.} \]

\[ L_v^* = \text{volume capacity of a vehicle in ft}^3 \]

Let \( f(m) \) denote the weekly distribution of miles \( \sim N(D\mu_r, \sqrt{D}\sigma_{LW}^2) \), which is dependent on \( N, R, \) and \( T \).

Let \( \Phi^{-1}(\cdot) \) represent the inverse Gaussian giving the miles associated with the cumulative probability contained within the parentheses.

Let \( f(N) \) denote the demand function. The number of delivery cycles determines how the weekly demand distribution breaks down for each cycle. For example, if the weekly demand distribution is normally distributed and \( D = 3 \), then \( f(N) \) equals the weekly demand mean divided by 3 and the expected daily demand has standard deviation equal to the square root of the weekly demand variance after being divided by 3. Thus, if the weekly demand mean is 100 customers and \( D = 4 \), then an average of 25 customers would be visited on each of the four delivery days. If \( D \) were changed to 5, then an average of 20 customers would be visited on each of the five delivery days.

Let \( p = \int_{m^*}^{\infty} f(m) dm \), the cumulative probability of the calculated midpoint, which is the point that bisects the probability of being greater than \( m^* \).

Let \( f(L_w) \) denote the distribution of the payload weight of individual customer orders

\[ \sim N(n\mu_{LW}, n\sigma_{LW}^2). \]

Let \( f(L_v) \) denote the distribution of the payload volume of individual customer orders

\[ \sim N(n\mu_{LV}, n\sigma_{LV}^2). \]

**Last Mile Delivery Fleet Planning Model Definition**

The total weekly cost of meeting customer demand, \( TC(T,D) \), is defined as the sum of maintenance cost, regular labor cost, overtime labor cost, fuel cost, and carbon emissions cost. This total cost function is as follows.

\[
TC(T,D) = C_mD \int_{N=1}^{\infty} \mu_r f(N) dN + C_wD \int_{N=1}^{\infty} \left( \frac{\mu_r}{V} + A(N-1) \right) f(N) dN
+ C_WP \left[ AD \int_{N=1}^{\infty} \left( \frac{\Phi^{-1}(p) - m^* \left( \int_{m^*}^{\infty} f(m) dm \right)}{N-1} \right) f(N) dN \right]
+ \int_{N=1}^{\infty} \left( \frac{\Phi^{-1}(p) - m^* \left( \int_{m^*}^{\infty} f(m) dm \right)}{V} \right) f(N) dN
+ (C_F + C_E)D \left( \frac{\int_{N=1}^{\infty} \mu_r f(N) dN}{\beta_0 + \beta_1 L \left( 1 - \frac{S}{2} \right) C_T} + A_B \beta_A \int_{N=1}^{\infty} (A(N-1)) f(N) dN \right)
\]

(1)

When examining the total cost equation (1), note that for a given radius of the demand region, \( R \), the expected distance traveled per day, \( \mu_T \), is a function of the number of vehicles, \( T \), and the node size, \( N \). The total cost equation defined in (1) represents the summation of five different costs: maintenance, regular labor, overtime labor, fuel, and carbon emissions.

Total maintenance cost in (1) is defined by multiplying maintenance cost per mile, \( C_m \), by expected distance traveled. The expected distance traveled is based on the number of delivery days,
$D$, multiplied by the distribution of expected demand, which depends on the number of delivery vehicles employed.

The total labor cost in (1) is defined by a summation of regular labor costs and overtime costs. In the first part, the labor wage, $C_w$, is multiplied by the number of delivery days, $D$, and then by the total labor hours expected per day. Calculating the total labor hours per day is based on the distribution of expected demand where the travel distance, $\mu_T$, is divided by the average vehicle speed, $V$, and then time spent at delivery stops, $A(N - 1)$, is added in.

The second part of calculating total labor costs involves an overtime cost, which applies if the weekly hours per vehicle exceed the regular wage time available. Overtime is composed of any time above the regular wage labor hours available per week spent at delivery stops and driving. The overtime cost associated with delivery stop time is estimated by finding the probability of exceeding $m^*$ for the week and then using the midpoint method (see Figure 1) to find the point at which that area is halved. That midpoint can be thought of as the expected total distance traveled when $m^*$ is exceeded. The midpoint is then used to calculate the average distance between nodes by dividing the midpoint by the number of nodes. The difference between the midpoint and the expected total weekly distance, $D\mu_T$, is then divided by this average distance between nodes to determine the approximate number of expected missed deliveries when $m^*$ is exceeded. Next, multiplying this value of the expected number of delivery stops that occur when $m^*$ is exceeded by the probability that $m^*$ is exceeded gives the overall number of expected overtime delivery stops. Finally, multiplying this overall number of expected overtime delivery stops by the average time spent at each stop gives the number of expected overtime hours spent at delivery stops per week.

The time spent driving is found similarly, but the difference between the midpoint and the total weekly distance, $D\mu_T$, is multiplied by the probability of exceeding $m^*$ and then divided by the average speed, $V$, to find the expected hours of overtime spent driving per week. These expected overtime hours are then multiplied by the labor wage premium since the regular wage rate has already been applied.

Figure 1: The Midpoint Used in the Calculation of Overtime

Figure 2 compares one parameterized example of the midpoint method of determining the expected number of overtime delivery stops with the computationally arduous method of calculating them for each level of demand individually. Since each number of customers has its own probability of occurring and within each of those, there is a probability associated with how often each would require overtime as well as the degree of overtime required, and calculating each individually is extensive. The midpoint method developed here is more computationally efficient and produces similar results.
The final component of (1) includes both the fuel cost and carbon emissions cost. According to Ülkü (2012), the carbon emissions cost is “a cost figure that is calibrated by management.” Zhang et al. (2015) model these costs similarly, allowing a decision maker to set these parameters as desired. See Chaabane et al. (2012) for an example of how a cost can be applied to total carbon emissions. Both fuel and carbon emission costs are based on the total gallons of fuel used and can be affected by average miles per hour, payload, time spent idling at delivery stops, and traffic congestion (Ülkü 2012). In equation (1), fuel cost, $C_F$, and carbon emissions cost, $C_E$, are multiplied by delivery days, $D$, and then by the expected gallons of fuel used per day, which includes both driving and idling at delivery stops. The expected gallons of fuel used while driving is determined by total mileage traveled per day divided by average fuel efficiency. Since fuel efficiency is affected by traffic congestion and payload weight, these factors and missed deliveries are taken into consideration to compute average fuel efficiency (Suzuki 2011; Xiao et al. 2012). Finally, fuel used while idling at delivery stops is calculated based on the likelihood of idling at a delivery stop multiplied by the gallons of fuel used per hour idling and then multiplied by the average time spent at delivery stops per day, which depends on the demand distribution.

Separating the calculation of total gallons of fuel used from (1), the total amount of weekly CO$_2$ emissions, $E(T, D)$, shown in (2), is defined as the amount of CO$_2$ emitted per gallon of fuel, $E_F$, multiplied by gallons of fuel used.

\[
E(T, D) = E_F D \left( \int_{N=1}^{\infty} \frac{\mu_T f(N) dN}{\beta_0 + \beta_1 L \left( 1 - \frac{S}{2} \right) C_T} + A_B \ln \left( \frac{A(N - 1)}{N - 1} \right) f(N) dN \right)
\]

From a managerial perspective, service level is another important consideration that must be addressed in addition to total cost when comparing alternative delivery options. For instance, if normal delivery hours were 8:00 AM to 6:00 PM, then delivering at 9:00 PM would not be desirable. This model allows a general delivery window to be set and will record the associated on-time delivery service levels. The on-time delivery service level, which is defined as the percentage of time that deliveries are made within the daily delivery window, is shown in (3).

\[
\frac{\int_0^m f(m) dm}{m}
\]
Payload weight and volume are captured as random variables and a service level is found for each delivery scenario. Similar to the on-time service level, a manager will choose relevant service levels for payload weight and volume in order to assure that the payload capacity of the vehicle is not exceeded. Service levels for daily payload weight and volume are defined in (4) and (5), respectively.

\[ (4) \int_0^{L_w} f(L_w) dL_w \]

\[ (5) \int_0^{L_v} f(L_v) dL_v \]

Model Illustration with a Numerical Example

In this section, the model is illustrated under a what-if scenario where the demand function is altered to simulate an expected spike in future customer demand. In this example, the company owns five delivery vehicles, which can adequately cover demand during peak times. However, demand is variable throughout the year. The following parameter values, modeled for a furniture store, have been assigned to support the model illustration:

- \( C_E = 0.02 \)
- \( C_F = 3.80 \)
- \( C_M = 0.05 \)
- \( C_W = 20 \)
- \( H = 12 \)
- \( H_W = 40 \)
- \( P = 0.5 \)
- \( R = 25 \)
- \( V = 30 \)
- \( A = 0.05 \)
- \( \beta_0 = 10 \)
- \( \beta_1 = -0.0005 \)
- \( \beta_A = 0.5 \)
- \( \beta_S = 0.5 \)
- \( \beta_T = 1 \)
- \( E_X = 10.18 \) kilos
- \( \mu_L = 80 \)
- \( \sigma_L = 15 \)
- \( \mu_LW = 10 \)
- \( \sigma_LW = 2 \)
- \( L_W = 4000 \)
- \( L_V = 500 \)
- \( f(N) \) is derived from weekly demand ~ \( N(300, 20) \).

The model was solved in an Excel spreadsheet. Probabilities from the demand distribution were applied to discrete values for the number of possible customers. Table 2 shows the results after removing any combinations of vehicles and delivery days resulting in service levels below 90% for available delivery hours and 95% for payload capacity (weight and volume). Based on the results, some managerial insight is needed to make the final delivery scenario decision based on the manager’s preference toward desired service levels. Thus, an absolute optimization is undesirable since providing multiple feasible options allows more insight and the flexibility to make informed decisions that meet customer needs and company objectives. In this example, the manager would likely conclude that using four vehicles delivering three days per week is the best possible option in terms of cost and acceptable service levels. The combinations of vehicles and delivery days with a service level below 95% have been italicized.
### Table 2: Results for Alternative Delivery Scenarios

<table>
<thead>
<tr>
<th>Vehicles</th>
<th>Days</th>
<th>Operating Cost</th>
<th>On-Time Service Level</th>
<th>Payload Weight Service Level</th>
<th>Payload Volume Service Level</th>
<th>CO₂ emissions (kilos)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>$2,387</td>
<td>90.2514%</td>
<td>99.4093%</td>
<td>99.3914%</td>
<td>2101</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$2,696</td>
<td>99.8671%</td>
<td>99.9995%</td>
<td>99.9994%</td>
<td>2284</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$2,987</td>
<td>99.9994%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2464</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$3,262</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2639</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$2,161</td>
<td>94.1135%</td>
<td>99.9986%</td>
<td>99.9984%</td>
<td>2039</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$2,434</td>
<td>99.9987%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2240</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$2,708</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2447</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$2,992</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2652</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$3,280</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2851</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>$2,189</td>
<td>99.9764%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2069</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$2,477</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2287</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$2,752</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2513</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$3,018</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2736</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$3,283</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2955</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>$2,354</td>
<td>99.9995%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2247</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$2,678</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2495</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$2,987</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2751</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$3,283</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>3005</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$3,571</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>3253</td>
</tr>
</tbody>
</table>

The responsiveness of the model to handling changes in customer demand is demonstrated by increasing weekly demand from $\sim \text{N}(300, 20)$ to $\sim \text{N}(360, 25)$ to simulate an increase in demand. Table 3 compares the results from each feasible scenario in Table 2 with this increase in demand. The combinations of vehicles and delivery days with a service level below 95% have been italicized to indicate that these options would likely be considered undesirable. As illustrated in Table 3, the former best choice of four vehicles delivering three days per week returns an on-time service level of just 92.56%. Therefore, the manager would need to change the delivery plan to avoid an unacceptable decrease in the service level resulting from the increase in demand. The best choice appears to be either a fleet of five vehicles delivering the same three days per week or the same fleet of four vehicles, but now delivering four days per week. The ultimate decision between the two delivery policies would come down to preferences among cost, delivery plan, and comfort level with the service levels since the four vehicles delivering four days per week is slightly more expensive and higher polluting, but offers a higher service level.
Table 3: Results for Alternative Delivery Scenarios

<table>
<thead>
<tr>
<th>Vehicles</th>
<th>Days</th>
<th>Operating Cost</th>
<th>On-Time Service Level</th>
<th>Payload Weight Service Level</th>
<th>Payload Volume Service Level</th>
<th>CO2 emissions (kilos)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>$2,786</td>
<td>45.8416%</td>
<td>80.6083%</td>
<td>80.5481%</td>
<td>2378</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$3,117</td>
<td>92.3368%</td>
<td>99.4259%</td>
<td>99.4117%</td>
<td>2558</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$3,427</td>
<td>99.6214%</td>
<td>99.9951%</td>
<td>99.9947%</td>
<td>2741</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$3,718</td>
<td>99.9900%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2920</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$2,425</td>
<td>45.9895%</td>
<td>98.0993%</td>
<td>98.0533%</td>
<td>2336</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$2,753</td>
<td>98.7236%</td>
<td>99.9998%</td>
<td>99.9998%</td>
<td>2525</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$3,092</td>
<td>99.9975%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2729</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$3,427</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2936</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$3,751</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>3141</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>$2,441</td>
<td>92.5563%</td>
<td>99.7496%</td>
<td>99.7496%</td>
<td>2361</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$2,747</td>
<td>99.9972%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2568</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$3,050</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2790</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$3,363</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>3015</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$3,681</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>3238</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>$2,617</td>
<td>98.5539%</td>
<td>99.7496%</td>
<td>99.7496%</td>
<td>2558</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$2,956</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>2793</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$3,282</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>3045</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$3,609</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>3301</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$3,942</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>100.0000%</td>
<td>3554</td>
</tr>
</tbody>
</table>

The methodology demonstrated presents a decision framework that managers could use for assessing the size and delivery plan of the vehicle fleet needed to satisfy customer demand in the context of a stochastic representation of the LMP. Clearly, the results demonstrated are parameter specific. Therefore, a sensitivity analysis was performed on the key parameters to better understand the features of the model.

Parameters associated with emissions, maintenance, and fuel costs ($C_E$, $C_M$, and $C_F$) were analyzed across a range with upper and lower bounds just outside of realistic values and did not change the resulting policy decision when changing individually or simultaneously. For the sensitivity analyses conducted, only the magnitude of the dollar difference between the solutions changed with all policy decisions remaining unchanged. Large changes to the labor wage rate per hour, $C_W$, however, did have an impact on the rank order of the solution set because of the effect on overtime cost.

Any parameters that impact expected overtime or service levels in some way will have an impact on the solution set. So changing $A$, $R$, $V$, $H$, and $H_W$ can all alter the feasibility of making the necessary deliveries in the allotted amount of time and, thus, affect the feasible solutions and the rank order of the solution set. Changes in these parameters within the feasibility range will only impact total cost and the solution set as they begin to incur overtime costs and then will ultimately be kicked out as the delivery option’s service level falls out of the acceptable range.

In addition, all parameters associated with the demand function, customer order weight and volume functions, and vehicle payload capacity (weight and volume) can strongly affect the service levels and, therefore, the feasible solutions and rank order of the solution set.

While the total cost function was found to not be convex, the sensitivity analysis shows that the parameters most responsible for impacting the optimal solution are those that affect the feasibility of making the required deliveries within the allotted time available for delivery. As these parameters change and constrict the feasibility and probability of successfully making the necessary deliveries,
certain delivery combinations become infeasible. The model provides decision makers with the ability to compare delivery alternatives in order to choose the delivery option with the best mix of cost, carbon emissions, and service level. Another contribution of this model is the model’s ability to highlight the differences in carbon emissions among decision alternatives that are similar in total cost and service level.

CONCLUSIONS

The last mile delivery fleet planning model presented herein defines a mathematical model that, for a given set of parameters, can be used to determine a decision maker’s preference in the combination of the number of vehicles and delivery days per week for delivering to end customers in a supply chain while taking on-time delivery and payload service levels into consideration. The framework presented extends distance-based models found in the literature by adopting a modeling structure that uniquely addresses the set of customer demand points found in the delivery region while also incorporating the cost of CO\textsubscript{2} emission into the model formulation. The model uniquely addresses the forward-looking nature of planning with uncertainty and adds to the literature by allowing a manager to perform what-if analyses to explore potential changes in delivery policy decisions based on cost, carbon emissions, and service levels by modifying inputs, such as the demand distribution, payload distributions, traffic congestion, fuel efficiency, time spent idling at delivery stops, and vehicle capacity.

A major contribution of the last mile delivery fleet planning model presented in this paper is incorporating stochastic demand into the LMP in a varying multiple vehicle delivery environment, incorporating carbon emissions that result from vehicle transportation into the LMP, and the development of a planning model that incorporates uncertainty in terms of expected mileage and labor hours. The last mile delivery fleet planning model advances the literature by creating a decision framework that includes a distribution of expected travel distance, probabilistic service levels, a carbon emissions component, and a multiple vehicle fleet of varying size. The combination of these factors in one delivery fleet planning model improves the quality of information available to plan properly, enhances overall decision-making, and adds to the literature with the ability to gauge the probabilistic likelihood of service levels and costs associated with solving the LMP. Managers can use the output from this model to explore fleet combinations as demand changes from week to week with more information than was previously available.

There are several aspects of the stochastic last mile delivery framework and the last mile delivery fleet planning model that can be extended. First, the model could be adapted such that carbon emission is constrained subject to a carbon trading/credit scheme. Second, the model could be expanded to include multiple depots within each region or sub region. Last, additional components including, but not limited to, vehicle purchase, depreciation cost, loading costs, labor burden, and/or driver salaries could be added to the model.

References


Fleet Planning


Jay R. Brown is an assistant professor of operations management at Loyola University Maryland. He received his Ph.D. in operations management from Kent State University. His research interests include operations research, stochastic modeling, carbon emissions, and last mile delivery. He has professional experience in operations and supply chain management. His research has been published in Information and Management, Journal of Manufacturing Systems, and International Journal of Logistics among other outlets.

Alfred L. Guiffrida is an associate professor of management and information systems at Kent State University. He holds his Ph.D. in industrial engineering from the University at Buffalo (SUNY). His research interests are in the areas of operations and supply chain management. He has published more than 30 articles in journals such as the European Journal of Operational Research, Industrial Marketing Management, International Journal of Production Economics, and the International Journal of Production Research.